Answers

(1) $\Delta ABC \sim \Delta DEF \Rightarrow \frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{1}{2} = \frac{8}{EF} \Rightarrow EF = \underline{16\,cm}$ ($\because 2AB = DE$).

(2) $12 \times \frac{4}{3}\pi r^3 = \pi . 8^2 . 2 \Rightarrow r = 2cm$. So diameter $= \underline{4cm}$.

(3) $AC = 10cm$. $BP = BQ = x$, $QC = CR = 6-x$, $AP = AR = 8-x$

So, ~~6+x+~~ $6-x+8-x = 10 \Rightarrow x = \underline{2cm}$.

(4) $2\pi r = 2r + 30 \Rightarrow \frac{44}{7}r - 2r = 30 \Rightarrow \frac{30r}{7} = 30 \Rightarrow r = \underline{7cm}$.

(5) A13.

(6) $S_{40} = 20\{2.2 + 39.4\} = 20 \times 160 = \underline{3200}$

or

$(a+17d) - (a+13d) = 32 \Rightarrow 4d = 32 \Rightarrow \underline{d = 8}$.

(7) $\frac{b+b+4}{2} = 1 \Rightarrow 2b + 4 = 2 \Rightarrow \underline{b = -1}$.

(8) $\cos 30^\circ = \frac{AT}{OT} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4} \Rightarrow \underline{2\sqrt{3}\,cm = AT}$

or

$\sqrt{13^2 - 5^2} = \underline{12cm}$.

(9) $25/35 = \underline{5/7}$

(10) $\underline{0}$

(11) mean $= \frac{5n}{9} \Rightarrow \frac{\left(\frac{n(n+1)}{2}\right)}{n} = \frac{5n}{9} \Rightarrow 9(n+1) = 10n \Rightarrow \underline{n = 9}$

(12) $G\left(\frac{-8+5+(-3)}{3}, \frac{0+5+(-2)}{3}\right) = \underline{G(-2, 1)}$

(13) $\frac{1}{3}\pi r^2 h = \pi r^2 6 \Rightarrow \underline{h = 18\,cm}$

(14) $\sin\theta = \cos\theta \Rightarrow \theta = 45^\circ$. So, $2\tan^2\theta + \cos^2\theta - 2 = 2 + \frac{1}{2} - 2 = \underline{\frac{1}{2}}$

or

$\tan 30^\circ = \frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{40} \Rightarrow \underline{40/\sqrt{3}\,cm = BC}$.

(15) $\alpha\beta^2 + \beta\alpha^2 = \alpha\beta(\beta+\alpha) = \frac{2}{3} \times \frac{-4}{3} = \underline{\frac{-8}{9}}$

or

$\alpha - \beta = 1 \Rightarrow \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 1 \Rightarrow \sqrt{(5)^2 - 4k} = 1 \Rightarrow 25 - 4k = 1 \Rightarrow \underline{4k = 24}$

(16) $96 = 2^5 \times 3$
$404 = 2^2 \times 101$, $HCF = 2^2 = \underline{4}$.

or

$\frac{17}{125} = \frac{17}{5^3} = \frac{17 \times 2^3}{2^3 \times 5^3} = \frac{17 \times 8}{(10)^3} = \frac{136}{10^3} = \underline{0.136}$.

(17) (a). (i) $\bar{x} \times 5$
(b) (ii) modal class
(c) (iii) 0.099
(d) (ii) 0.08 – 0.12
(e) (ii) 8

(18) (a) (ii) $50\sqrt{3}$ m
(b) (i) 150 m
(c) (i) $100\sqrt{3}$ m
(d) (ii) 100 m
(e) (ii) decreasing.

(19) (a) (ii) $1/3$
(b) (i) $2/3$
(c) (ii) $1/6$
(d) (iv) none of these
(e) (iv) 100%.

(20) (a) (i) 12 cm
(b) (iv) none of these
(c) (i) 0
(d) (iv) none of these
(e) (ii) chord.

(21) Reqd distance = LCM(40, 42, 45) cm = 2520 cm.

(22) $2.\left(\frac{5}{2}\right)^2 - 8\left(\frac{5}{2}\right) - k = 0 \Rightarrow \underline{k = -15/2}.$

Equation reduces to $2x^2 - 8x + \frac{15}{2} = 0$
$\Rightarrow 4x^2 - 16x + 15 = 0 \Rightarrow 4x^2 - 10x - 6x + 15 = 0 \Rightarrow (2x-5)(2x-3) = 0$
So, the other root is $\underline{3/2}$

or

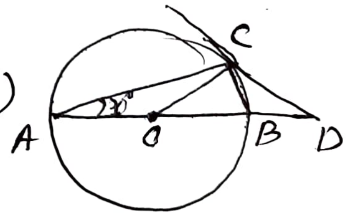
$\underline{x = a/3 \text{ or } b/3}$.

(23) $\underline{2:1}$

(24) $PT = PS \Rightarrow \angle T = \angle S = 60°$. So, PTS is an equilater Δ. So $\underline{TS = 4\text{cm}}$

Infinite no of lines of the length as TS can be drawn.

or



Proof. $\angle OAC = \angle OCA = 30°$
$\angle OCB = 90° - 30° = 60°$ ($\because$ angle in semi circle.)
$\angle BCD = 90° - 60° = 30°$ (radius $\perp$ tangent)
Now, in Δ ACD, $\angle D = 180° - (30° + 120°) = 30°$
Clearly, $\angle BCD = \angle BDC$.
So, BC = BD (Proved)

(25) LHS:- $\cos 3\theta = \cos 3.30° = \cos 90° = 0$
RHS:- $4\cos^3\theta - 3\cos\theta = 4.\cos^3 30° - 3\cos 30° = 4.\left(\frac{\sqrt{3}}{2}\right)^3 - 3.\frac{\sqrt{3}}{2}$
$= 4.\frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2} = 0.$
So, $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (Proved)

(26) Original no = $10.3x + x = 31x$.
New no = $10.x + 3x = 13x$.
ATQ = $31x - 13x = 54 \Rightarrow x = 3$.
So, required no is 93. (Ans)

(27) $a(q-r)+b(r-p)+c(p-q)$

$= a_p(q-r)+a_q(r-p)+a_r(p-q)$

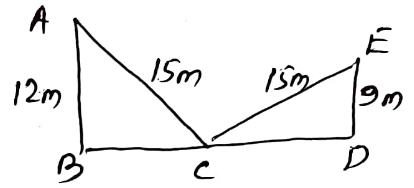
$=\{a+(p-1)d\}(q-r)+\{a+(q-1)d\}(r-p)+\{a+(r-1)d\}(p-q)$

$=a\{q-r+r-p+p-q\}+d\{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)\}$

$= 0+d\{\cancel{pq}-\cancel{pr}-\cancel{q}+\cancel{r}+\cancel{qr}-\cancel{qp}-\cancel{r}+\cancel{p}+\cancel{rp}-\cancel{rq}-\cancel{p}+\cancel{q}\}$

$= 0$ (Proved)

(28) AC & CE are the two positions of the ladder.



So, $AC=CE=15m$.

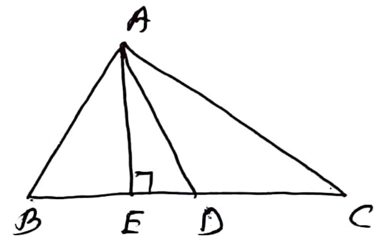
$AB=12m$, $DE=9m$.

Clearly, $BC=\sqrt{15^2-12^2}=9m$.

$CD=\sqrt{15^2-9^2}=12m$.

Width of street $=9+12=\underline{21m}$

or



In Δ ABC, AD is a median.

Let's draw $AE\perp BC$.

LHS. $AB^2+AC^2 = AE^2+BE^2+AE^2+CE^2$ ($\because AE\perp BC$)

$=2AE^2+BE^2+CE^2$

$=2(AD^2-DE^2)+(BD-DE)^2+(CD+DE)^2$

$=2AD^2-\cancel{2DE^2}+BD^2+\cancel{DE^2}-\cancel{2BD.DE}+CD^2+\cancel{DE^2}+\cancel{2CD.DE}$

$=2AD^2+2BD^2$ ($\because BD=CD$)

$=2(AD^2+BD^2)$. (Proved)

(29) Area of design = Area of 2 quadrants − Area of square

$=2\times\frac{1}{4}.\pi.8^2-8^2$

$=\left(\frac{1}{2}.\frac{22}{7}-1\right)8^2=\frac{4}{7}.64=\frac{256}{7}\ cm^2.$

(30) Proof.

(31) $PR=PQ \Rightarrow \sqrt{(a+b-x)^2+(a-b-y)^2}=\sqrt{(b-a-x)^2+(a+b-y)^2}$

$\Rightarrow \cancel{a^2}+\cancel{b^2}+\cancel{x^2}+\cancel{2ab}-\cancel{2bx}-2ax+\cancel{a^2}+\cancel{b^2}+\cancel{y^2}-\cancel{2ab}+2by-\cancel{2ay} = \cancel{b^2}+\cancel{a^2}+\cancel{x^2}-\cancel{2ab}+2ax-\cancel{2bx}+\cancel{a^2}+\cancel{b^2}+\cancel{y^2}+\cancel{2ab}-2by-\cancel{2ay}$

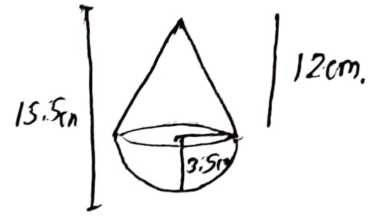
$\Rightarrow 2by-2ax=2ax-2by$

$\Rightarrow 4by=4ax \Rightarrow by=ax$ (Proved)

(32) or $\tan A = \sqrt{3} \Rightarrow A = 60^\circ$.
But $B = 90^\circ \Rightarrow C = 30^\circ$.

Now, $\sin A \cos C + \cos A \sin C = \sin 60^\circ \cos 30 + \cos 60^\circ \sin 30$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$



(33) For the hemispher $r = 3.5$cm.
For the cone $r = 3.5$cm
$h = 12$cm
$\Rightarrow l = 12.5$cm,

$TSA = \pi r l + 2\pi r^2 = \pi r (l + 2r)$

$= \frac{22}{7} \times 3.5 (12.5 + 2 \times 3.5) = \frac{22}{7} \times \frac{35}{10} \times \frac{195}{10} = 214.5 \text{ cm}^2$

$V = \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 (h + 2r) = \frac{1}{3} \cdot \frac{22}{7} \cdot \frac{35}{10} \times \frac{35}{10} (12 + 7)$

$= 243.83 \text{ cm}^3$

(34) $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x} \Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow (x+a)(x+b) = 0 \Rightarrow x = -a, -b$.

(35) Constn.

(36) $\alpha + \beta = -b/a$, $\alpha\beta = c/a$.

$s' = \alpha' + \beta' = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -b/c$. $p' = \alpha'\beta' = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c}$

$k[x^2 - s'x + p'] = k[x^2 + \frac{b}{c}x + \frac{a}{c}] = k[acx^2 + abx + a^2]$

$= acx^2 + abx + c^2$ (for $k = ac$). is the reqd poly. (A)

So, the reqd poly is $\underline{cx^2 + bx + c}$ (A)

or

$\alpha^2 + \beta^2 + \alpha\beta = (\alpha+\beta)^2 - 2\alpha\beta + \alpha\beta = (\alpha+\beta)^2 - \alpha\beta$

$= \left(\frac{-5}{2}\right)^2 + \frac{k}{2} = \frac{25 - 2k}{4}$.

ATQ, $\frac{25 - 2k}{4} = \frac{21}{4} \Rightarrow$ ~~50 - 4k = 21~~ $\underline{k = 2}$.