SECTION - A

(1) Smallest odd composite number is 9.

(2) Let the required ratio be K : 1

So $A\left(\frac{2K+1}{K+1}, \frac{7K+3}{K+1}\right)$ is a point on $3x+y-9=0$

$\Rightarrow 3\left(\frac{2K+1}{K+1}\right)+\left(\frac{7K+3}{K+1}\right)-9=0$

$\Rightarrow 6K+3+7K+3-9K-9=0$

$\Rightarrow 4K-3=0$

$\Rightarrow K=\frac{3}{4}$

$\therefore$ The required ratio is 3 : 4.

(3) The parabola must be opening up or down.

4) $\frac{h_1}{h_2}=\frac{1}{4}$ ; $\frac{r_1}{r_2}=\frac{4}{1}$

$\frac{V_1}{V_2}=\frac{\cancel{1/3\pi} r_1 h_1}{\cancel{1/3\pi} r_2 h_2}=\left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right)=\left(\frac{4}{1}\right)^2\left(\frac{1}{4}\right)=\frac{4}{1}=4:1$

(5) $8\tan\theta=15$

$\tan\theta=\frac{15}{8}$

(8, 15 & 17 are pythagorean triplet)

$\sec\theta=\frac{17}{8}$ $\cos\theta=\frac{8}{17}$ $\sin\theta=\frac{15}{17}$

So, $\sin\theta-\cos\theta=\frac{15}{17}-\frac{8}{17}=\frac{7}{17}$

(6) -7 ~~OR~~

OR

$a_n = a_{54}+132$

$\cancel{3}+(n-1)12=\cancel{3}+53\times12+132$

$n-1=\frac{53\times12+132}{12}=\frac{768}{12}=64 \Rightarrow n=64+1 \Rightarrow n=65$

07) $\frac{\cancel{6}}{\cancel{360}_{\cancel{60}_{30}}} \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \cancel{7} \times 7 = \frac{77}{30} cm^2 = 2.56 cm^2$

OR

$\frac{1}{\cancel{3}}\cancel{\pi}r_1^2 h + \frac{1}{\cancel{3}}\cancel{\pi}r_2^2 h = \frac{4}{\cancel{3}}\cancel{\pi}R^3$

$\Rightarrow (r_1^2 + r_2^2)h = 4R^3 \quad \Rightarrow h = \frac{4R^3}{r_1^2 + r_2^2}$

(8) Mode = 3 median - 2 mean

$\Rightarrow$ 45 = 3 median - 2 × 27

$\Rightarrow$ 45 + 54 = 3 median

$\Rightarrow$ median = $\frac{45+54}{3} \Rightarrow \frac{99}{3}$

$\Rightarrow$ median = 33

(9) Road distance = $\sqrt{3^2 + 4^2} = 5$ cm

(10) $\frac{AB}{PQ} = \frac{\text{Perimeter } \Delta ABC}{\text{Perimeter } \Delta PQR} \Rightarrow \frac{5.5}{11} = \frac{70}{\text{Perimeter } \Delta PQR}$

$\Rightarrow$ Perimeter $\Delta PQR = \frac{11 \times 70}{5.5}$

$\therefore$ Perimeter of $\Delta PQR$ is 140 cm.

OR

$R = \sqrt{10^2 + 7^2} = \sqrt{149}$ cm

(11) 0

12) $\tan^2 45 - \cos^2 30 = x \sin 45° \cdot \cos 45°$

$1 - \left(\frac{\sqrt{3}}{2}\right)^2 = x \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$

$\Rightarrow 1 - \frac{3}{4} = x\frac{1}{2} \Rightarrow \frac{1}{4} \times 2 = x \Rightarrow x = \frac{1}{2}$

OR

$\sin A = \frac{2}{5} \Rightarrow \operatorname{cosec} A = \frac{5}{2}$

$\Rightarrow \cot^2 A = \operatorname{cosec}^2 A - 1 = \frac{25}{4} - 1 \Rightarrow \frac{21}{4}$

Now,

$5 + \cancel{4} \times \frac{21}{\cancel{4}} \Rightarrow 5 + 21 \Rightarrow 26$

(13) $\frac{BA}{BC} = \frac{QA}{QP} \Rightarrow \frac{6}{8} = \frac{QA}{4} \Rightarrow QA = \frac{6 \times 4}{8} \Rightarrow QA = 3cm$

(14) $2(2+3+4) = 2 \times 9 = 18$ cm

(15) $a\sqrt{2} = 6\sqrt{2} \Rightarrow a = 6$ cm

(16) $\frac{\sin 45°}{\sec 30 + \operatorname{cosec} 30} = \frac{1/\sqrt{2}}{\frac{2}{\sqrt{3}} + \frac{2}{1}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})}$

OR

LHS, $\sin 2A = \sin 2 \times 0° = \sin 0° = 0$

RHS, $2 \sin A = 2 \sin 0° = 2 \times 0 = 0$

Clearly LHS = RHS

So for $A = 0°$ $\sin 2A = 2 \sin A$.

17 (a) (i) Parabolic

(b) (ii) 3, 4

(c) (ii) $x^2 + 2x - 8$

(d) (iii) 4

(e) (i) 2.

18 (a) (ii) 62

(b) (ii) 12

(c) (iii) median

(d) (i) 55

(e) (ii) 55

19 (a) (ii) ~~Isosceles~~ equilateral

(b) (i) 8 cm

(c) (iii) 5 m

(d) (i) 9.43 cm

(e) (iv) $16\sqrt{3}$ m

20 (a) (i) (2,3), (6,6)

(b) (ii) 5

(c) (i) 3

(d) (ii) $(10/3, 3)$

(e) (iii) 12 sq. units.

21. $10224 = 2^4 \times 3^2 \times 71$

$1608 = 2^3 \times 3 \times 67$

LCM $= 2^4 \times 3^2 \times 71 \times 67$

$= 6,85,008$

HCF $= 2^3 \times 3$

$= 24$

22. $(5\sqrt{3})^2 - 4 \times 2 \times 6$

$= 75 - 48$

$= 27 > 0$

Hence, real roots exist.

$2x^2 + 4\sqrt{3}x + \sqrt{3}x + 6 = 0$

$2x(x + 2\sqrt{3}) + \sqrt{3}(x + 2\sqrt{3}) = 0$

$(2x + \sqrt{3})(x + 2\sqrt{3}) = 0$

So the roots are

$x = -\frac{\sqrt{3}}{2}$ & $x = -2\sqrt{3}$.

23. Modal class = 40-50

$l = 40$, $f_1 = 20$, $f_0 = 12$, $f_2 = 11$ $h = 10$

Mode $= 40 + \left(\frac{20-12}{40-12-11}\right) 10 \Rightarrow 40 + \left(\frac{8}{17} \times 10\right) \Rightarrow 40 + \frac{80}{17} \Rightarrow 40 + 4.7$

$\Rightarrow 44.7$

OR

| class | frequency | $x_i$ | $f_i x_i$ |
|---|---|---|---|
| 0-20 | 17 | 10 | 170 |
| 20-40 | P | 30 | 30p |
| 40-60 | 32 | 50 | 1600 |
| 60-80 | 24 | 70 | 1680 |
| 80-100 | 19 | 90 | 1710 |
| | 92+P | | 5160+30p |

$\Rightarrow 50 = \frac{5160+30p}{92+P} \Rightarrow 4600+50p = 5160+30p$

$\Rightarrow 50p - 30p = 5160 - 4600 \Rightarrow 20P = 560$

$\Rightarrow P = \frac{\cancel{560}\,28}{\cancel{20}} \Rightarrow P = 28$

$\therefore$ The required value of P is 28.

24) LHS $= \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$

$\Rightarrow \frac{\sin^2 A \cos^2 B - \sin^2 B \cos^2 A}{\cos^2 A \cos^2 B} = \frac{(1-\cos^2 A)\cos^2 B - (1-\cos^2 B)\cos^2 A}{\cos^2 A \cos^2 B}$

$\Rightarrow \frac{\cos^2 B - \cancel{\cos^2 A \cos^2 B} - \cos^2 A + \cancel{\cos^2 A \cos^2 B}}{\cos^2 A \cos^2 B}$

$\Rightarrow \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$

Also, $\frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} = \frac{1-\sin^2 B - (1-\sin^2 A)}{\cos^2 A \cos^2 B} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

$\Rightarrow \tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B} = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B}$

(Hence proved)

OR

$\sqrt{3} \tan 2\theta - 3 = 0$

$\Rightarrow \sqrt{3} \tan 2\theta = 3$

$\Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$\Rightarrow 2\theta = 60^\circ \quad = \theta = 30^\circ$

$\therefore \cos 30^\circ$ is $\frac{\sqrt{3}}{2}$

(25) Let $AM = MB = x$ (given)

$PN = NQ = y$

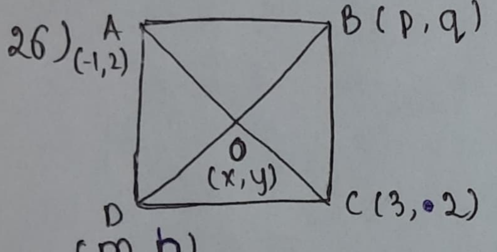
$\frac{x}{y} = \frac{AM}{PN}$ ; $\frac{AM}{PN} = \frac{AB}{PQ} = \frac{CA}{RP}$ $\quad (\because \triangle ABC \sim \triangle PQR)$

So, $\frac{AM}{PN} = \frac{CA}{RP}$ & $\angle A = \angle P$

Therefore $\triangle AMC \sim \triangle PNR$ (SAS similarity criteria)

(Hence proved)

26)




$AB = BC$

$= \sqrt{(p+1)^2+(q-2)^2} = \sqrt{(p-3)^2+(q-2)^2}$

$\Rightarrow (p+1)^2 + \cancel{(q-2)^2} = (p-3)^2 + \cancel{(q-2)^2}$

$\Rightarrow \cancel{p^2} + 2p + 1 = \cancel{p^2} - 6p + 9$

$\Rightarrow 2p + 1 = 9 - 6p$

$\Rightarrow 8p = 8 \Rightarrow \boxed{p = 1}$

In right $\triangle ABC$,

$AC^2 = 2AB^2$

$(3+1)^2 + (2-2)^2 = 2[(p+1)^2 + (q-2)^2]$

$\frac{16}{2} = (1+1)^2 + (q-2)^2 \quad (p=1)$

$\Rightarrow 8 = 4 + q^2 + 4 - 4q \Rightarrow 8 - 8 = q^2 - 4q \Rightarrow 0 = q^2 - 4q$

$\Rightarrow 0 = q(q-4) \Rightarrow q - 4 = 0 \Rightarrow \boxed{q = 4}$ & $\boxed{q = 0}$

The coordinates of O $\left(\frac{-1+3}{2}, \frac{2+2}{2}\right) \Rightarrow O(1, 2)$

The coordinates of O considering BD $= \left(\frac{1+m}{2}, \frac{q+b}{2}\right)$

$(1, 2) = \left(\frac{1+m}{2}, \frac{q+b}{2}\right)$

$\Rightarrow \frac{1+m}{2} = 1 \Rightarrow m = 2 - 1 \Rightarrow m = 1$

$2 = \frac{q+b}{2}$ $\Rightarrow 4 = q+b$ when $q = 0$, $4 = 0+b \Rightarrow b = 4$

when $q = 4 \Rightarrow 4 - 4 = b \Rightarrow \boxed{b = 0}$

$\therefore$ The coordinate of B is (1, 4) or (1, 0) & the coordinate of D is (1, 4) or (1, 0).

27) Let the three angles be

$a,\ a+d,\ a+2d$

$\Rightarrow a+2d = a+a+d$

$\Rightarrow a+2d = 2a+d$

$\Rightarrow a = d$

By ASP of triangle

$a+a+d+a+2d = 180$

$3a+3d = 180$

$a+d = 60$

$2a = 60 \quad (\because a = d)$

$a = 30$

$\Rightarrow d = 30$

$\therefore$ The ~~two~~ three angles are $30°$, $60°$ & $90°$

OR

Amount = 1000

Interest = $1000 \times \frac{8}{100} \times 1 = 80$ 3rd year = $3 \times 1000 \times \frac{8}{100} = 240$

2nd year = $1000 \times \frac{8}{100} \times 2 = 160$

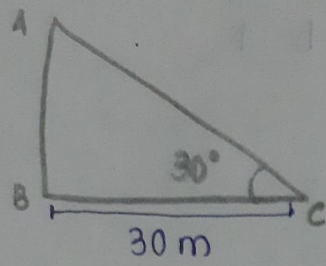
$\Rightarrow 240 - 160 = 160 - 80$

$\Rightarrow 80 = 80$

Thus they form an AP

$\therefore$ Interest on the end of 30 year will be $1000 \times \frac{8}{100} \times 30 = ₹\ 2{,}400$

28.




In right $\Delta$ ABC,

$\tan 30^\circ = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30} \Rightarrow AB = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow AB = \frac{30\sqrt{3}}{3} \Rightarrow AB = 10\sqrt{3} \Rightarrow AB = 17.3$ m

Thereforce, the tree bent a height at 17.3m

Now, In right $\Delta$ ABC,

$\cos 30^\circ = \frac{BC}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{30}{AC} \Rightarrow AC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow AC = \frac{60\sqrt{3}}{3} \Rightarrow AC = 20\sqrt{3}$

$AC = 34.6$ m

$\therefore$ The total height of the tree was $34.6 + 17.3$ m $= 51.9$m

29. $R = \frac{80}{2} = 40$ cm

1 complete Revolution = circumference

$C = 2 \times \pi \times 40 = 80\pi$ cm $= 0.8\pi$ m

Speed $= 66$ Km/hr $= \frac{66 \times 1000}{60} = 1100$ m/min

The car will travel a distance of $= 1100 \times 10 = 11000$ m

No. of Revolution $= \frac{11000 \times 7}{0.8 \times 22} = 4375$

$\therefore$ The car will make 4375 complete revolutions.

30. let $a = c$

$\Rightarrow a + \sqrt{b} = c + \sqrt{d}$ (given)

$\Rightarrow \cancel{c} + \sqrt{b} = \cancel{c} + \sqrt{d}$

$\Rightarrow \sqrt{b} = \sqrt{d}$

Squareing both sides

$b = d$

if $a = c$ then $b = d$

Let $a = c + k$ (where k is any +ve reational no.)

$a + \sqrt{b} = c + \sqrt{d}$

$\Rightarrow \cancel{c} + k + \sqrt{b} = \cancel{c} + \sqrt{d}$

$\Rightarrow k + \sqrt{b} = \sqrt{d}$

Squareing both sides

$K^2 + b + 2K\sqrt{b} = d$

$\sqrt{b} = \frac{d - K^2 - b}{2K}$

It is given that, b, d & K are rational no.

$\Rightarrow \frac{d - K^2 - b}{2K}$ = rational $\Rightarrow \sqrt{b}$ = rationional.

Now,

$(\sqrt{b})^2$ = (rational)$^2$

b = rational$^2$

$K + \sqrt{b} = \sqrt{d}$

rational + rational = $\sqrt{d}$

$\Rightarrow \sqrt{d}$ = rational

$(\sqrt{d})^2$ = (rational)$^2$

$\Rightarrow$ d = rational$^2$

31. (i) Total outcome = 36

Favoureable outcome = 5

Probability = $\frac{5}{36}$

(ii) Total outcome = 36

Favoureable outcome 0

Probobility = $\frac{0}{36} = 0$

(iii) Total outcome = 36

Favoureable outcome 36

Probability = $\frac{\cancel{36}}{\cancel{36}} = 1$

OR

(i) Total outcome = 52 - 8 = 44

Favorable outcome = 2

probabily = $\frac{\cancel{2}^{1}}{\cancel{44}_{22}} = \frac{1}{22}$

(ii) Total out come = 44

Favoureable outcome = 18

Probability = $\frac{18}{44}$

(iii) Total outcome: 44

Favorable outcome: 4

Probability: $\frac{\cancel{4}\ 1}{\cancel{44}\ 11} = \frac{1}{11}$

32. $5x - y = 5$

| $x$ | 0 | 1 |
|---|---|---|
| $y$ | -5 | 0 |

$3x - y = 3$

| $x$ | 0 | 1 |
|---|---|---|
| $y$ | -3 | 0 |

Therefore, $\Delta ABC$ is the required triangle where $A(1,0)$, $B(0,-3)$, $C(0,-5)$

33. $\frac{(1 + \cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \operatorname{cosec}^3 A} = \sin^2 A \cdot \cos^2 A$

$\Rightarrow \frac{\left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)}{\frac{1}{\cos^3 A} - \frac{1}{\sin^3 A}}$

$\Rightarrow \frac{\frac{(\sin A \cos A + \cos^2 A + \sin^2 A)(\sin A - \cos A)}{\sin A \cos A}}{\frac{\sin^3 A - \cos^3 A}{\sin^3 A \cos^3 A}}$

$\Rightarrow \frac{\sin^3 A \cancel{+} - \cos^3 A}{\sin A \cos A} \times \frac{\sin^3 A \cos^3 A}{\sin^3 A - \cos^3 A}$

$\Rightarrow \frac{\sin^3 A \cos^3 A}{\sin A \cos A} \Rightarrow \sin^2 A \cos^2 A = RHS$

(Hence proved)

34. Let the no. of marbles with Ram is $x$

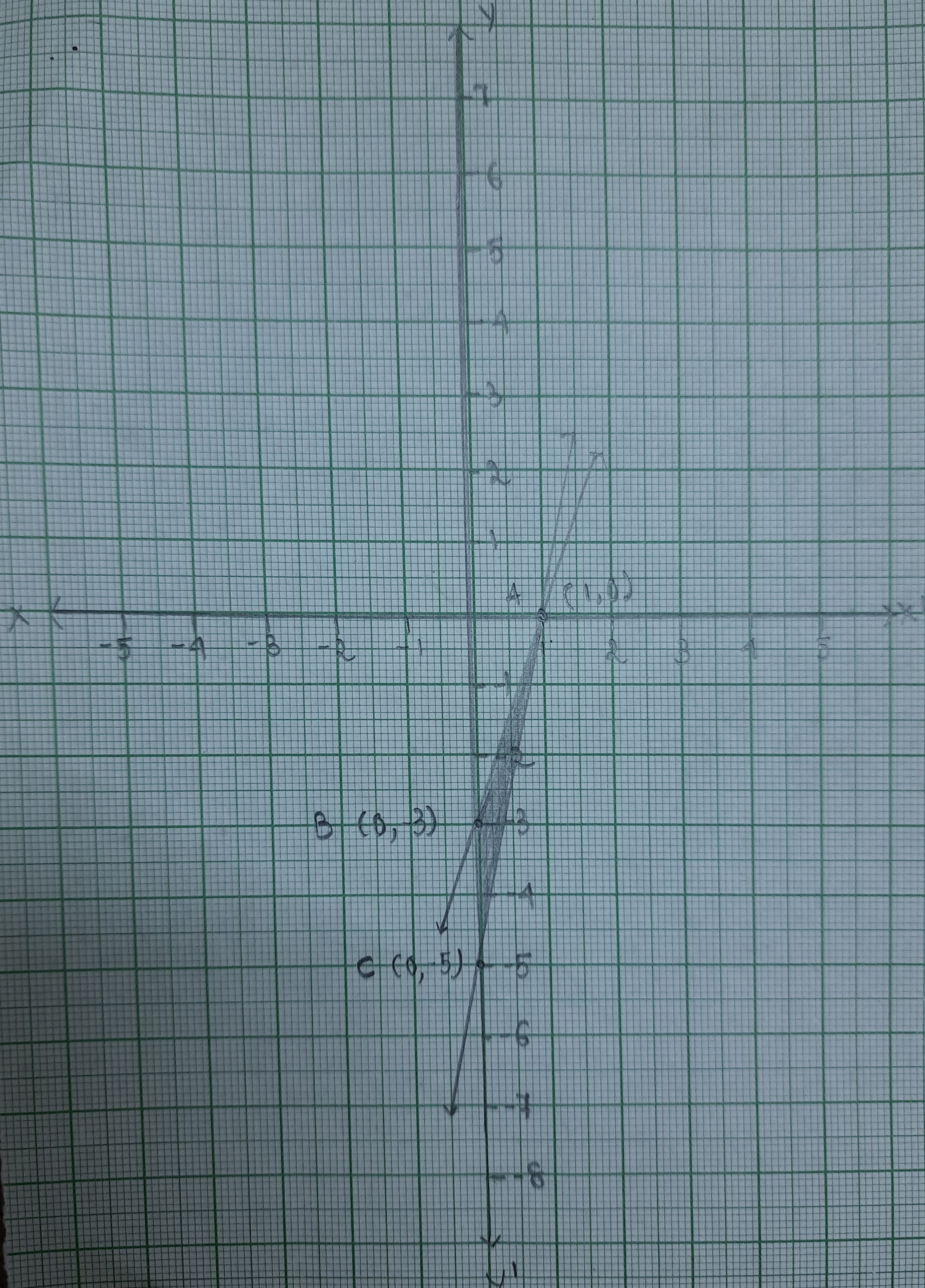
No. of marbles with Shyam: $55 - x$

As both of them lose ~~one~~ 5 marbles, then the no. of marbles with Ram is $x - 5$

Shyam is $55 - x - 5 = 50 - x$

ATQ,

$(x - 5)(50 - x) = 164$



Y
7
6
5
4
3
2
1
A (1, 0)
X
X'
-5
-4
-3
-2
-1
1
2
3
4
5
B (0, -3)
C (0, -5)
-6
-7
-8
Y'

$\Rightarrow 50x - x^2 - 250 + 5x = 164$

$\Rightarrow -x^2 + 55x - 250 - 164 = 0$

$\Rightarrow -x^2 + 55x - 414 = 0$

$\Rightarrow x^2 - 55x + 414 = 0$

$\Rightarrow x^2 - 46x - 9x + 414 = 0$

$\Rightarrow x(x-46) - 9(x-46) = 0$

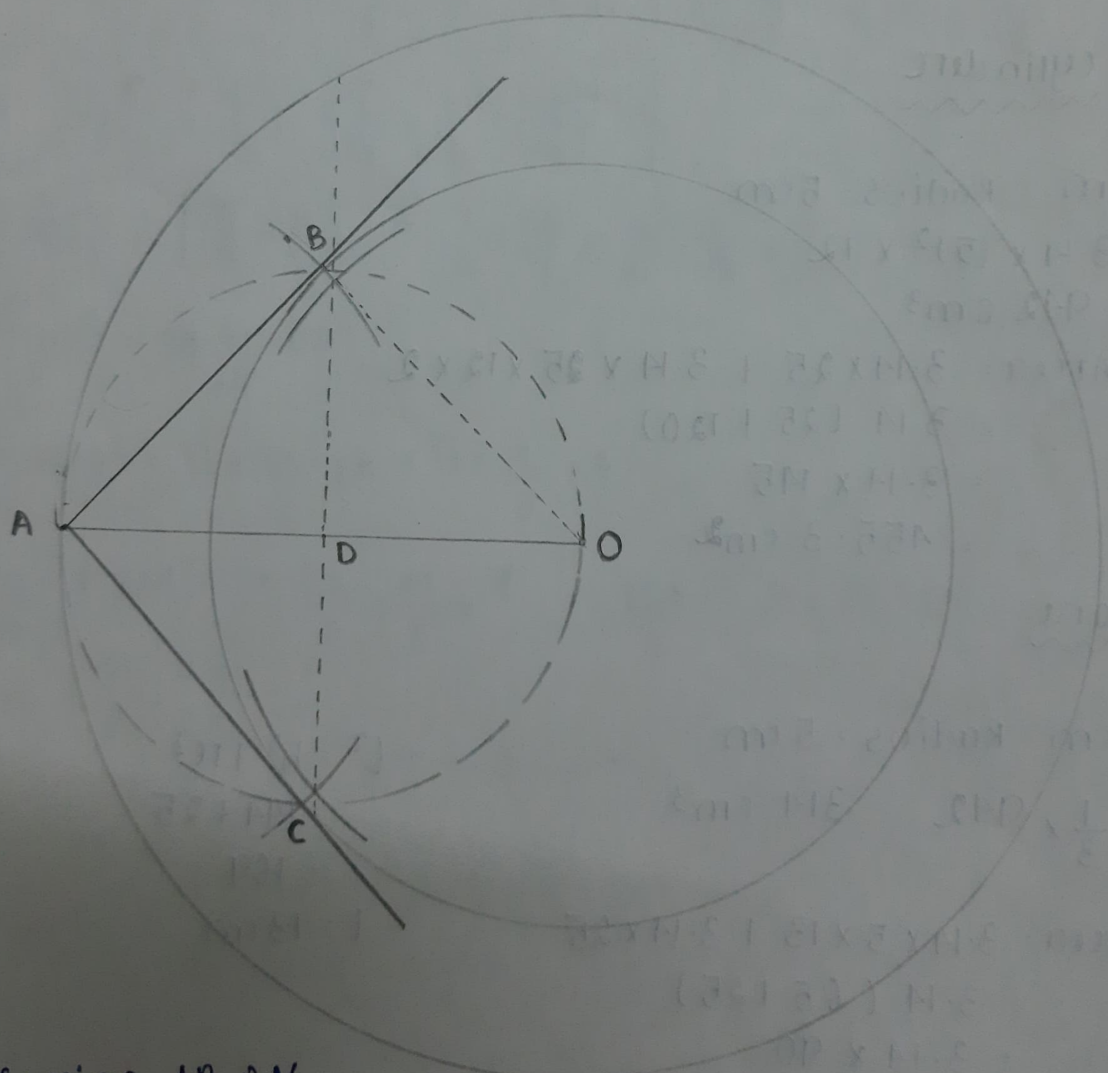
$\Rightarrow (x-9)(x-46) = 0$

$\Rightarrow x - 9 = 0$ & $x - 46 = 0$

$\Rightarrow x = 9$ & $x = 46$

$\therefore$ when Ram will have 46 marbles, then Shyam will have 9 & when Ram will have 9 marbles then Shyam will have 46.

35.




On measuring AB & AC,

AB = AC = 4.8 cm

$\Delta$ ABO is a right $\Delta$, right angled at B.

By pythagoras thm,

$OB^2 + AB^2 = OA^2$

$AB^2 = 7^2 - 5^2$

$= 49 - 25$

$= 24$

$AB = \sqrt{24} = 4.8$ cm

(Hence verified)

Steps of construction:

* Two circles of radii 5cm & 7cm are drawn from one centre
* ⊥ of of OA is drawn which intersect OA at D.
* Taking OD as radius and D as centre a circle is drawn.
* It intersects the circle of radius 5cm at B & C.
* AB & AC are joined and hence are the required tangents.

36. For the cylinder

height = 12cm  Radius = 5cm

volume = $3.14 \times (5)^2 \times 12$

$= 942$ $cm^3$

Surface Area = $3.14 \times 25 + 3.14 \times 25 \times 12 \times 2$

$= 3.14\ (25 + 120)$

$= 3.14 \times 145$

$= 455.3$ $cm^2$

For the cone

height = 12cm  Radius = 5cm

volume = $\frac{1}{3} \times 942 = 314$ $cm^3$

$l^2 = h^2 + r^2$

$= 144 + 25$

$= 169$

$l = 13$cm

Surface Area = $3.14 \times 5 \times 13 + 3.14 \times 25$

$= 3.14\ (65 + 25)$

$= 3.14 \times 90$

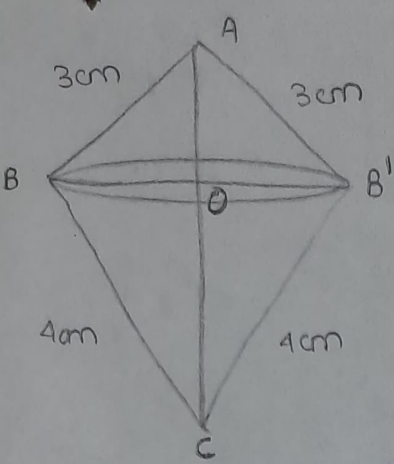
$= 282.6$ $cm^2$

∴ volume of of the remaining solid = $942 cm^3 - 314 cm^3$

$= 628$ $cm^3$

... of remaining Solid = $455.3 - 282.6$ $cm^2$

$= 172.7$ $cm^2$




OR

In right $\Delta ABC$,

$\sqrt{3^2 + 4^2} = AC$

$\Rightarrow AC = 5cm$

Now, In $\Delta AOB$ & $\Delta ABC$,

$\angle AOB = \angle ABC$ ($90^\circ$ each)

$\angle A = \angle A$ (common)

$\Delta AOB \sim \Delta ABC$ (AA Similarity)

$\Rightarrow \frac{AB}{AC} = \frac{OB}{BC} = \frac{OA}{AB} \Rightarrow \frac{3}{5} = \frac{OB}{4} \Rightarrow OB = \frac{12}{5}$

$\Rightarrow \frac{AB}{AC} = \frac{OA}{AB} \Rightarrow \frac{3}{5} = \frac{OA}{\cancel{AB}\,3} \Rightarrow OA = \frac{9}{5}$

$\Rightarrow AC = AO + OC \Rightarrow 5 = \frac{9}{5} - OC \Rightarrow OC = \frac{25-9}{5} \Rightarrow OC = \frac{16}{5}$

volume $= \frac{1}{3}\pi \left(\frac{12}{5}\right)^2 \times \frac{9}{5} + \frac{1}{3}\pi \left(\frac{12}{5}\right)^2 \frac{16}{5}$

$\Rightarrow \frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 = \left[\frac{9}{5} + \frac{16}{5}\right] \Rightarrow \frac{1}{\cancel{3}} \times \frac{22}{7} \times \frac{\cancel{12}^{4}}{\cancel{5}} \times \frac{12}{\cancel{5}} \times \frac{\cancel{25}}{5}$

$\Rightarrow \frac{1056}{35} cm^3 = 30\frac{6}{35} cm^3$

Surface Area $= \pi \times \frac{12}{5} \times 3 + \pi \times \frac{12}{5} \times 4$

$\Rightarrow \frac{22}{7} \times \frac{12}{5} (3+4) \Rightarrow \frac{22}{\cancel{7}} \times \frac{12}{5} \times \cancel{7} = \frac{264}{5} = 52.75 \, cm^2$.