Solutions

1. Given, $V(x) = \frac{20}{(x^2-4)}$ V

As, $E = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{20}{x^2-4}\right)$

$= +20\,(x^2-4)^{-2} \times 2x = \frac{40x}{(x^2-4)^2}$ [1/2]

At $x = 4\,\mu$m,

$E = \frac{40 \times 4}{(16-4)^2} = \frac{160}{144} = \frac{10}{9}$ V/μm [1/2]

2. Since, charge (q) = current (i) × time (t)

Therefore, charge is equal to area under the i-t curve.

∴ Charge through interval 1s to 2s, $q = lb = 2$ [1/2]

Charge through interval 2s to 3s, $q = lb = 2$

Charge through interval 3s to 5s, $q = \frac{1}{2}lb = 2$

Hence, ratio is 1 : 1 : 1. [1/2]

3. The induced emf is expected to be constant only in the case of the rectangular loop. In the case of circular loop, the rate of change of area of the loop during its passage out of the field region is not constant, hence induced emf will vary accordingly. [1]

Or

The polarity of plate A will be positive with respect to plate B in the capacitor. Both magnets, by motion, increase rightward magnetic flux through coil, so induced emf with produce leftward flux and positive charge will come at A and negative on B. [1]

4. The intensity of principal maximum in the single slits diffraction pattern does not depend upon the slit width. Thus, the intensity will remain saime. [1]

Or

Distance upto which ray optics is a good approximation is Fresnel's distance Z_f,

where, $Z_f = \frac{a^2}{\lambda} = \frac{(3 \times 10^{-3})^2}{5 \times 10^{-7}} = 18$ m [1]

5. The image formed in a microscope at least distance of distinct vision ($D = 25$ cm) has magnification

$M = M_o \times M_e$

where, M_o is magnification of objective lens and M_e that of eyepiece.

Also, $M = M_o\left(1 + \frac{D}{f_e}\right)$ [1/2]

Given, $f_e = 5$ cm, $D = 25$ cm and $M = 30$

$30 = M_o \times \left(1 + \frac{25}{5}\right)$

$M_o = \frac{30}{6} = 5$ [1/2]

Or

As we know,

$\delta = (\mu - 1)A$

Here, $\mu = 1.5$

$A = 6°$

$\Rightarrow \delta = (1.5 - 1) \times 6 = 3°$ [1]

6. On doping, there is an additional energy level (called donor level in case of pentavalent impurities and acceptor level in case of trivalent impurities) which gets added between valence and conduction bands. Hence, energy gap reduces. [1]

Or

When a diode is reverse biased, the p-side is connected to negative terminal and the n-side is connected to positive terminal of battery. This reduces the drift current (due to majority carriers). Hence, the potential barrier in depletion region increases. [1/2]

As the drift current due to majority carriers becomes very small, the effective resistance offered to the flow of carriers becomes very large. [1/2]

7. As we know that, electric field inside a conductor is always zero. Therefore, the electric field lines drawn by the student inside the metallic sphere are inappropriate. [1]

8. Given, speed, $v = 1800$ km/h $= 500$ m/s

Length of the span of wings, $l = 25$ m

Earth's magnetic field, $B = 5 \times 10^{-4}$ T

Angle of dip, $\delta = 30°$

Vertical component of earth's magnetic field,

$B_V = B\sin\delta$

$= 5 \times 10^{-4} \times \frac{1}{2}$

$= 2.5 \times 10^{-4}$ T [1/2]

(∵ Only the vertical component of earth's field will cut horizontally moving plane.)

Induced emf, $E = B_V lv = 2.5 \times 10^{-4} \times 25 \times 500 = 3.1$ V [1/2]

9. Kinetic energy of photoelectrons is given by

$K_{max} = \frac{1}{2}mv_{max}^2 = h\nu - \phi_0$

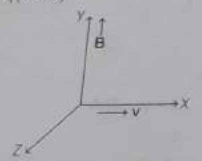
$= \frac{hc}{\lambda} - \phi_0$ [1/2]

Since, h, c and ϕ_0 are constants, so we can write

$\Rightarrow \quad v_{max}^2 \propto \frac{1}{\lambda}$ or $v_{max} \propto \frac{1}{\sqrt{\lambda}}$

Thus, as the wavelength of incident light decreases, the velocity of photoelectrons increases. [1/2]

10. Since, $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$



So, $\mathbf{v} \times \mathbf{b} = \hat{i} \times \hat{j} = \hat{k}$

(i) So, using right hand thumb rule, force on electron will be along $-Z$-axis. [1/2]

(ii) Force on a positive charge or proton will be along $+Z$-axis. [1/2]

11. (a) The maximum amount of charge a capacitor can have depends on the shape and size of capacitor and also on the surrounding medium.

Thus, a capacitor can be given only a limited quantity of charge.

Therefore, A and R are true and R is the correct explanation of A. [1]

12. (b) On increasing the temperature of a conductor, the kinetic energy of free electrons increases.

On account of this, they collide more frequently with each other (and with the ions of the conductor) and consequently their drift velocity decreases.

So, on increasing temperature, conductivity of metallic wire decreases.

Therefore, A and R are true and R is not the correct explanation of A. [1]

13. (a) A stationary charge produces electric field only. However, a moving charge which is equivalent to a current is produce a magnetic field in the surrounding space and can interacts with external magnetic field.

Therefore, A and R are true and R is the correct explanation of A. [1]

14. (a) In n-type semiconductor, the pentavalent dopant is donating one extra electron for conduction.

Thus, the number of electrons will now be due to the electrons contributed by donors and those generated intrinsically. However, holes will only be due to the intrinsic source.

So, the rate of recombination of holes would increase due to the increase in number of electrons. As a result, the number of holes would get reduced further.

Therefore, A and R are true and R is the correct explanation of A. [1]

15. (i) (a) As nearly 99.9% mass of atom is in nucleus.

$\therefore \quad \frac{\text{Mass of nucleus}}{\text{Mass of atom}} = \frac{99.9}{100} = 0.99 \approx 1$ [1]

(ii) (a) Since, the nuclei of deuterium and tritium are isotopes of hydrogen, they must contain only one proton each. But the masses of the nuclei of hydrogen, deuterium and tritium are in the ratio of 1 : 2 : 3, because of presence of neutral matter in deuterium and tritium nuclei. [1]

(iii) (d) Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$

$$= \frac{3m}{4\pi R_0^3}$$

As, $\quad m = m_P = m_N$

$= 2.3 \times 10^{17}$ kgm^{-3}, which is a constant. [1]

(iv) (a) $R = R_0 A^{1/3}$

$$\log R = \log R_0 + \frac{1}{3}\log A$$

On comparing the above equation of straight line, $y = mx + c$. So, the graph between $\log A$ and $\log R$ is a straight line also. [1]

(v) (a) Here $A_1 = 197$ and $A = 107$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{197}{107}\right)^{1/3} = 1.225$$

$$\approx 1.23$$ [1]

16. (i) (b) The wavelength of visible light is very small, that is hardly shows diffraction, so it seems to propagate in rectilinear path.

(ii) (c) Angular width of central maxima,

$$2\theta = 2\lambda / e$$

Thus, θ does not depend on D i.e., distance between the slit and the screen. [1]

(iii) (c) The direction in which the first minima occurs is θ (say.)

Then, $e\sin\theta = \lambda$ or $e\theta = \lambda$ or $\theta = \frac{\lambda}{e}$

($\because \theta = \sin\theta$, when θ is small)

Width of the central maxima $= 2b\theta + e = \frac{2\lambda b}{e} \pm e$ [1]

(iv) (b) As, the path difference $a\theta$ is λ,

Then $\quad \theta = \frac{\lambda}{a} \Rightarrow \frac{10\lambda}{d} = \frac{2\lambda}{a}$)

$\Rightarrow \quad a = \frac{d}{5} = \frac{10}{5} = 0.2$ mm

So, the width of each slit is 0.2 mm. [1]

(v) (a) Width of central maxima $= 2\lambda D / e$

width of other secondary maxima $= \lambda D / e$

$\therefore$ width of central maxima : width of other secondary maxima $= 2:1$ [1]

17. Work done to take charges is independent of path followed. Therefore,

$$W = Q\left(\frac{q}{4\pi\varepsilon_0 r_2} - \frac{q}{4\pi\varepsilon_0 r_1}\right)$$

$$= \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

Here, $q = 8$mC $= 8 \times 10^{-3}$C [1]

$Q = -2 \times 10^{-9}$C

$r_1 = 3 \times 10^{-2}$cm

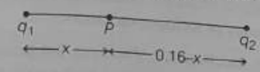
$r_2 = 4 \times 10^{-2}$m

$\Rightarrow \quad W = -2 \times 10^{-9} \times 8 \times 10^{-3} \times 9 \times 10^9 \times \left[\frac{1}{4 \times 10^{-2}} - \frac{1}{3 \times 10^{-2}}\right]$

$= 12$ J [1]

Or

Let the two charges be placed as shown in figure



Let potential be zero at point P between the two charges at distance x from q_1.

$$V_{q_1} + V_{q_2} = 0$$ [1]

where, V_{q_1} is potential due to q_1

and V_{q_2} is potential due to q_2.

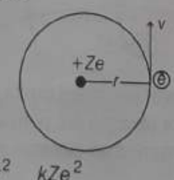
$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{0.16 - x}\right] = 0$

or $\quad 9 \times 10^9\left[\frac{5 \times 10^{-8}}{x} - \frac{3 \times 10^{-8}}{0.16 - x}\right] = 0$

$\Rightarrow \quad x = 10$ cm [1]

18. Radius of nth Bohr orbit

To keep electron in orbit, centripetal force equal to electrostatic force



Therefore, $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$

where, $\quad k = \frac{1}{4\pi\varepsilon_0}$

$\Rightarrow \quad r = kZe^2 / mv^2$ ...(i)

where, m is the mass of the electron and v its speed in an orbit of radius r.

Bohr's quantisation condition for angular momentum is

$$mvr = \frac{nh}{2\pi}$$

$\Rightarrow \quad r = \frac{nh}{2\pi mv}$ ...(ii) [1]

From Eqs. (i) and (ii), we get

$$\frac{kZe^2}{mv^2} = \frac{nh}{2\pi mv}$$

$\Rightarrow \quad v = \frac{2\pi k Ze^2}{nh}$

Putting this value of v in Eq. (ii), we get

$$r = \frac{nh}{2\pi m} \cdot \frac{nh}{2\pi k Ze^2} = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

or $\quad r \propto n^2$ [1]

Or

(i) Yes, they are the isotopes of the same element because they have same atomic number ($Z = 3$). [1/2]

(ii) The isotope $_3^7X$ has 3 protons and 4 neutrons while the isotopes $\frac{4}{3}Y$ has 3 protons and 1 neutron. Due to the presence of a greater number of neutrons in $_3^7X$, the strong attractive nuclear force dominates over the electrostatic repulsion between the protons, so $\frac{7}{3}X$ is more stable than $\frac{4}{3}Y$. [1½]

19. Here, net emf of circuit,

$$E = E_2 - E_1 = 9 - 5 = 4 \text{ V}$$

and total resistance of the circuit,

$$R = \frac{6 \times 3}{6 + 3} + 4.5 + 0.3 + 1.2 = 8\,\Omega$$

$\therefore$ Main circuit current, $I = E / R = 4\text{ V} / 8\,\Omega = 0.5$ A [1]

If current flowing through $3\,\Omega$ resistance be I_1, then current flowing through $6\,\Omega$ resistance will be $(0.5 - I_1)$ and hence

$$3I_1 = 6 \times (0.5 - I_1)$$

$\Rightarrow \quad I_1 = 0.33$ A

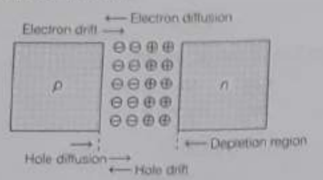
So, the current flowing through $3\,\Omega$ resistance is 0.33 A. [1]

20. The two main processes taking place during the formation of p-n junction are diffusion and drift.

Diffusion is due to majority carriers in which holes diffuse from p to n-side and electrons diffuse n to p-side.

Due to recombination near junction region, a layer of immobile charge carriers called depletion layer is formed. This set-up an electric field in the junction region. [1]

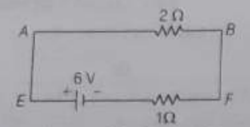
Due to field in junction region, there is drift of minority carriers, i.e. electrons drift from p to n-side while holes drift from n to p-side. This process of drift of minority carriers is called drift.



[1]

Or

Here, D_1 is forward biased and D_2 is reverse biased, so the circuit becomes



Equivalent resistance, $R = 2 + 1 = 3\,\Omega$ [1]

$$I_{EF} = \frac{V}{R} = \frac{6}{3} = 2\,\text{A}$$ [1]

21. (i) **Electric field intensity** Electric field intensity at any point due to some charge is defined as the force experienced by a unit positive charge placed at that point.

i.e. $\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}$

where, $\mathbf{E}$ = electric field intensity,
$\mathbf{F}$ = force
and q_0 = small test charge.
$\mathbf{E}$ is a vector quantity.
Its SI unit is Newton/Coulomb or (NC^{-1}) [1]

(ii) **Electric flux** Total number of field lines passing through a given area when the area is held normal to the field is called electric flux. Flux through an area $d\mathbf{S}$ due to electric field $\mathbf{E}$ at an angle θ is

$$d\phi = \mathbf{E} \cdot d\mathbf{S} = EdS\cos\theta$$

It is a scalar quantity.
Its SI unit is Nm^2C^{-1}. [1]

22. The linear width of central maximum is given by

$$\beta = \frac{2D\lambda}{d}$$

(i) If monochromatic yellow light is replaced with red light, the linear width of the central maximum increases because $\lambda_{red} > \lambda_{yellow}$ [1]

(ii) If the distance (D) between the slit and screen is increased, the linear width of the central maximum increases. (as $\beta \propto D$) [1]

23. (i) γ-rays are produced by radioactive decay of the nucleus. [1]

(ii) Since, we know that, the energy of an electromagnetic wave, $E \propto \nu$ (frequency)

$\because \nu_{\text{visible light}} < \nu_{\text{ultraviolet rays}} < \nu_{\gamma\text{-rays}}$

Thus, $E_{\text{visible light}} < E_{\text{ultraviolet rays}} < E_{\gamma\text{-rays}}$ [1]

24. (i) In photoelectric effect, the saturation current does not vary with anode potential for incident radiations of different frequencies, but same intensity. The reason is that saturation current depends only on intensity of incident radiation (because a single photon can eject a single electron) and not the frequency, however large the frequency of radiations may be. [1]

(ii) According to the Einstein's photoelectric equation, the stopping potential (V_0) is given by

$$eV_0 = h\nu - \phi_0$$

or $$V_0 = \frac{h\nu}{e} - \frac{\phi_0}{e}$$

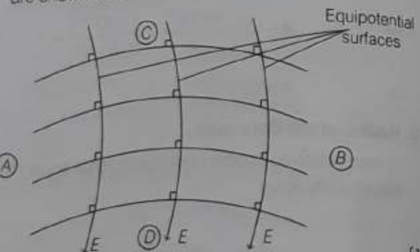
Obviously, stopping potential is independent of intensity, i.e. stopping potential does not vary with intensity of incident radiations. It only depends on frequency of incident radiation. [1]

25. (i) Due to electrostatic shielding, the person in the car is not affected (as electric field inside the metallic body is zero). [1]

(ii) Since, we know that, the electric field lines are perpendicular to equipotential surfaces and

$$E = -\frac{dV}{dr}$$ [1/2]

Negative sign implies that electric potential drops in the direction of electric field. So, the equipotential surfaces are shown below as



[1/2]

26. Bohr's postulate of permitted orbits is that, only those circular orbits for electron are permitted in which angular momentum of an electron is an integral multiple of $h/2\pi$.

i.e. $$mvr = n\frac{h}{2\pi}$$ ...(i) [1]

where, n is an integer.

From de-Broglie hypothesis, wavelength associated with electron, $\lambda = \frac{h}{mv}$

$\Rightarrow$ $$mv = \frac{h}{\lambda}$$ [1]

Substituting this value in Eq. (i), we get

$$\frac{h}{\lambda} r = n\frac{h}{2\pi} \text{ or } 2\pi r = n\lambda$$

This shows that the circumference of the nth orbit contains exact n de-Broglie wavelengths. [1]

Or

The energy of gaseous hydrogen at room temperature are as given below

$$E_1 = -13.6\text{ eV},\ E_2 = -3.4\text{ eV}$$
$$E_3 = -1.51\text{ eV},\ E_4 = -0.85\text{ eV}$$
$$E_3 - E_1 = -1.51 - (-13.6) = 12.09\text{ eV}$$ [1]

and $E_4 - E_1 = -0.85 - (-13.6) = 12.75$ eV

As, both the values do not match the given value, but it is nearest to $E_4 - E_1$.

$\therefore$ Upto $E_4 - E_1$ energy level, the H-atoms would be excited.

Lyman series, $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$

For first member, $n = 2$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 1.097 \times 10^7 \left[\frac{4-1}{4}\right]$$

$$\Rightarrow \quad \lambda_1 = 1.215 \times 10^{-7}\text{ m}$$ [1]

Balmer series, $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$

For first member, $n = 3$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \quad \lambda_1 = 6.56 \times 10^{-7}\text{ m}$$ [1]

27. (i) Given, $E_{rms} = 200$ V, $L = 5$ H,
$C = 80\,\mu F = 80 \times 10^{-6}$ F and $R = 40\,\Omega$

(a) For the maximum current in the circuit, $X_L = X_C$

$$\Rightarrow \quad \omega L = \frac{1}{\omega C}$$

$\therefore$ Resonant frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}}$$

$$= 50\text{ rad/s}$$ [1]

(b) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$Z = R = 40\,\Omega \quad [\because X_L = X_C]$$

Current, $$I_{rms} = \frac{E_{rms}}{Z} = \frac{200}{40} = 5\text{A}$$

Current amplitude at resonance, $I_0 = I_{rms}\sqrt{2}$

$$= 5 \times 1.414 = 7.07\text{ A}$$ [1]

(ii) It is given that, a voltage is applied across parallel LC. Since, current in L lags behind voltage by 90° phase, current in C leads voltage by 90° phase. So, current in L and C are 180° out of phase. [1]

Or

(i) Given, $E = (100\sin 314t)$ V

As the current in a capacitor leads the voltage by 90°, so the instantaneous current is given by

$$I = I_0\sin(314t + 90°) = I_0\cos 314t$$

where, $$I_0 = \frac{E_0}{X_C} = \frac{E_0}{1/\omega C} = E_0\omega C$$

But, $E_0 = 100$ V, $\omega = 314$ rad s^{-1}, $C = 637 \times 10^{-6}$ F

$$\therefore \quad I_0 = 100 \times 314 \times 637 \times 10^{-6} = 20\text{ A}$$

Hence, $I = 20\cos 314t$ ampere. [2]

(ii) Angular frequency of power, $\omega_p = 628$ rad s^{-1}

$\therefore$ Frequency of power, $f_H = \frac{\omega_H}{2\pi} = \frac{628}{2\pi} = 100$ Hz [1/2]

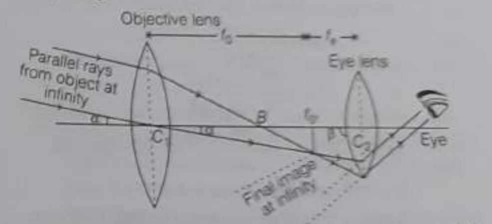
(iii) The maximum energy stored in the capacitor is

$$U_0 = \frac{1}{2}CE_0^2 = \frac{1}{2} \times 637 \times 10^{-6} \times (100)^2 = 3185\text{ J}$$ [1/2]

28. Magnifying power (M) of a telescope point when final image is formed at infinity is

$$M_\infty = -f_o/f_e$$ [1]

Ray diagram

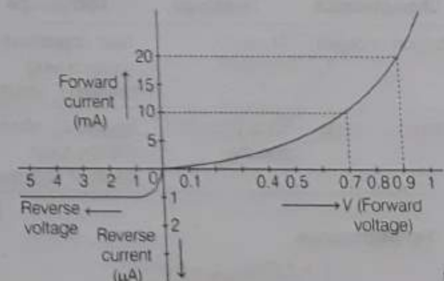


[1]

Here, f_o is the focal length of objective lens and f_e is the focal length of eyepiece.

The diameter of objective is kept large to increase the (i) intensity of image (ii) and resolving power of telescope. [1]

29. (i) The forward and reverse characteristics can be drawn as



[1½]

(ii) Considering diode characteristics to be almost a straight line between 10 mA and 20mA, using Ohm's law, resistance can be calculated.

(a) From forward bias characteristic, at $I = 10$ mA, $V = 0.7$ V and

at $I = 20$ mA, $V = 0.9$ V

$$\Rightarrow r_{fb} = \frac{\Delta V}{\Delta I} = \frac{0.9-0.7}{(20-10)}$$

$$= \frac{0.2}{10\text{mA}} = 0.2 \times 100 = 20\Omega$$

(b) From the reverse bias curve, at

$V = -4$V, $I = -1\mu$A

Therefore, $r_{rb} = \frac{V}{I} = \frac{4\text{V}}{1\mu\text{A}} = 4 \times 10^{6}\Omega$ [1½]

30. (i) Given, $n = 20$, $E = 2$V, $r = 0.5\Omega$ and $R = 10\Omega$

If all cells are connected correctly in series to the load R, by Shahana then

$$I = \frac{nE}{R+nr} = \frac{20 \times 2}{10 + 20 \times 0.5}$$

$$= \frac{40}{10+10} = \frac{40}{20} = 2\text{A}$$

It justifies the set-up of Shahana. [1]

(ii) If one cell is wrongly connected in series circuit, then it will reduce the total emf of the circuit by the two times of its own emf. Let m cells are connected wrongly by Shikha, then we have

$$I_1 = \frac{(n-2m)E}{R+nr}$$

$$\Rightarrow 1.2 = \frac{(20-2 \times m)2}{10+20 \times 0.5}$$

$$\Rightarrow (20-2m) = \frac{1.2 \times (10+10)}{2} \Rightarrow 20-2m = 12$$

$$\therefore m = (20-12)/2 = 4$$

It means, 4 cells are connected wrongly by Shikha. [1½]

(iii) For maximum current, $R = 0$

$$\Rightarrow I_{max} = \frac{E}{r} = \frac{20}{0.5} = 40\text{A}$$ [1/2]

31. Differences between telescope and microscope are given as below

Characteristics	Telescope	Microscope
Position of object	At infinity	Near objective at a distance lying between f_o and $2f_o$
Position of image	Focal plane of objective	Beyond $2f_o$ when f_o is the focal length of objective

[2]

For microscope

$f_o = 1.25$ cm, $f_e = 5$ cm

When final image forms at infinity, then magnification produced by eye lens is given by

$$M = -\frac{L}{f_o} \cdot \frac{D}{f_e} \Rightarrow -30 = -\frac{L}{1.25} \times \frac{25}{5}$$

$$L = \frac{30 \times 1.25}{5} \Rightarrow L = 7.50 \text{ cm}$$ [1]

For objective lens

$v_o = L = 7.5$ cm

$f_o = 1.25$ cm, $u_o = ?$

Applying lens formula,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\Rightarrow \frac{1}{1.25} = \frac{1}{7.5} - \frac{1}{u_o}$$

$$\frac{1}{u_o} = \frac{1}{7.5} - \frac{1}{1.25}$$

$$= \frac{1.25 - 7.5}{7.5 \times 1.25} = -\frac{6.25}{7.5 \times 1.25}$$ [1]

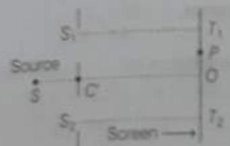
$$\Rightarrow u_o = -\frac{7.5 \times 1.25}{6.25}$$

$$= -1.5 \text{ cm}$$

The object must be at a distance of 1.5 cm from objective lens. [1]

Or

(i)



According to the arrangement, by geometry

$T_2P = T_2O + OP = D + x$

and $T_1P = T_1O - OP = D - x$

$$S_1P = \sqrt{(S_1T_1)^2 + (PT_1)^2} = \sqrt{D^2 + (D-x)^2}$$

and $S_2P = \sqrt{(S_2T_2)^2 + (T_2P)^2} = \sqrt{D^2 + (D+x)^2}$

The minima will occur when $S_2P - S_1P = (2n-1)\frac{\lambda}{2}$

i.e., $[D^2 + (D+x)^2]^{1/2} - [D^2 + (D-x)^2]^{1/2} = \frac{\lambda}{2}$

[for first minima $n = 1$] [2]

If $x = D$

we can write $[D^2 + 4D^2]^{1/2} - [D^2 + 0]^{1/2} = \frac{\lambda}{2}$

$$\Rightarrow [5D^2]^{1/2} - [D^2]^{1/2} = \frac{\lambda}{2}$$

$$\Rightarrow \sqrt{5}D - D = \frac{\lambda}{2}$$

$$\Rightarrow D(\sqrt{5}-1) = \frac{\lambda}{2} \Rightarrow D = \frac{\lambda}{2(\sqrt{5}-1)}$$

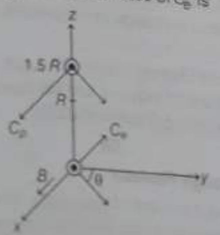
Putting $\sqrt{5}-1 = 2.236 - 1 = 1.236$

$$\Rightarrow D = \frac{\lambda}{2(1.236)} = 0.404\lambda$$ [2]

(ii) To observe interference fringe pattern, there is need to have coherent sources of light so that they can produce light of constant phase difference. [1]

32. Since, B is along the X-axis, for a circular orbit the momenta of the two particles will be in the yz-plane. Let p_1 and p_2 be the momentum of the electron and positron, respectively. Both traverse a circle of radius R in opposite direction w.r.t. each other. Let p_1 make an angle θ with the Y-axis, p_2 must make the same angle. [2]

The centres of the respective circles must be perpendicular to the momenta and at a distance R. Let the centre of the electron be at C_e and of the positron at C_p. The coordinates of C_e is



$$C_e = (0, -R\sin\theta, R\cos\theta)$$

The coordinates of C_p is, $C_p = (0, -R\sin\theta, \frac{3}{2}R - R\cos\theta)$

The circles of the two shall not overlap, if the distance between the two centres are greater than $2R$.

Let d be the distance between C_p and C_e.

Then, $d^2 = (2R\sin\theta)^2 + \left(\frac{3}{2}R - 2R\cos\theta\right)^2$

$$= 4R^2\sin^2\theta + \frac{9}{4}R^2 - 6R^2\cos\theta + 4R^2\cos^2\theta$$

$$= 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta$$

Since, d has to be greater than $2R$, $d^2 > 4R^2$

$$\Rightarrow 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta > 4R^2$$

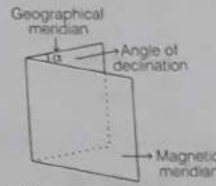
$$\Rightarrow \frac{9}{4} > 6\cos\theta$$

or $$\cos\theta < \frac{3}{8}$$ [3]

Or

Elements of Earth's magnetic field The Earth's magnetic field at a place can be described by three parameters known as elements of Earth's magnetic field. They are declination, dip and horizontal component of Earth's magnetic field. [1/2]

(a) **Magnetic declination** The angle between the geographical meridian and the magnetic meridian at a given place is called the magnetic declination (α).



[1/2]

(b) **Angle of dip or magnetic inclination** The angle made by the Earth's total magnetic field B with the horizontal direction in the magnetic meridian is called angle of dip (δ) at that place. [1/2]

[1/2]

For different places on Earth, there is different angle of dip.

(c) **Horizontal component of Earth's magnetic** It is the component of the Earth's total magnetic field (B) in the horizontal direction in the magnetic meridian. If δ is the angle of dip at any place, then the horizontal component of Earth's field B at that place is given by

$$B_H = B\cos\delta$$

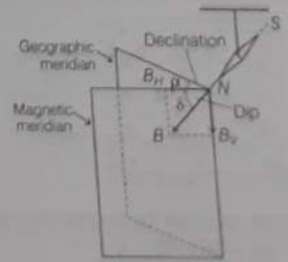
At the magnetic equator, $\delta = 0°$,

$$B_H = B\cos 0° = B$$

At the magnetic poles, $\delta = 90°$,

$$B_H = B\cos 90° = 0$$ [1/2]

Thus, the value of B_H is different at different places on the surface of the Earth.



[1/2]

Relations between elements of Earth's magnetic figure shows the three elements of Earth's magnetic field. If δ is the angle of dip at any place, then the horizontal and vertical components of Earth's magnetic field **B** at that place will be

$$B_H = B\cos\delta \quad \text{...(i)}$$

and $$B_V = B\sin\delta$$

$$\therefore \quad \frac{B_V}{B_H} = \frac{B\sin\delta}{B\cos\delta}$$

or $$\frac{B_V}{B_H} = \tan\delta \quad \text{...(ii)}$$

Also, $$B_H^2 + B_V^2 = B^2(\cos^2\delta + \sin^2\delta) = B^2$$

or $$B = \sqrt{B_H^2 + B_V^2} \quad \text{...(iii)}$$

Eqs. (i), (ii) and (iii) are the different relations between the elements of Earth's magnetic field. By knowing the three elements, we can determine the magnitude and direction of the Earth's magnetic field at any place. [2]

33. (i) Since, according to Faraday's law, the emf induced in a conductor whenever magnetic flux through it changes is given by

$$\varepsilon = -N\frac{d\phi}{dt}$$

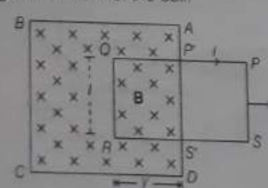
where, N = Number of turns of coil (conductor)
and ϕ = flux through the conductor
But $\phi = BA\cos\theta$
where, B = magnetic field,
A = area of conductor
and θ = angle between **B** & **A** (Area vector)

$$\Rightarrow \quad \varepsilon = -N\frac{d}{dt}(BA\cos\theta)$$

Thus emf can be induced by
(a) Changing the number of turns of coil, N
(b) Change the intensity of magnetic field, B
(c) Changing the area linked with field, A
(d) Changing the orientation of coil, θ [2]

(ii) **Motional electromotive force and faraday's law**

Consider a uniform magnetic field **B** confined to the region $ABCD$ and a coil $PQRS$ is placed inside the magnetic field. At any time t, the part $P'Q = S'R = y$ of the coil is inside the magnetic field. Let l be the length of the arm of the coil.



Area of the coil inside the magnetic field at time t,

$$\Delta S = QR \times RS' = ly$$

Magnetic flux linked with the coil at any time t,

$$\phi = B\Delta S = Bly$$

The rate of change of magnetic flux linked with the coil is given by

$$\frac{d\phi}{dt} = \frac{d}{dt}(Bly)$$

$$= Bl\frac{dy}{dt} = Blv \qquad \left[\because \frac{dy}{dt} = v\right]$$

where, v is the velocity with which the coil is pulled out of the magnetic field.

If e is the induced emf, then according to Faraday's law,

$$e = -\frac{d\phi}{dt}$$

or $$e = -Blv$$ [2]

(iii) Polarity of induced emf can be given by Lenz's law. According to Lenz's law, the polarity of induced emf is such that, it tends to produce a current which opposes the change in magnetic flux that produced it.

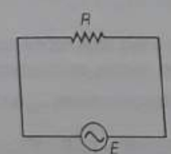
Also from Fleming's right hand rule, the induced current due to induced emf will flow from the end R to Q, i.e. along $SRQP$ in the coil. [1]

Or

(i) **AC through Resistor**

Suppose a resistor of resistance R is connected to an AC source of emf with instantaneous value (E) which is given by

$$E = E_0\sin\omega t \quad \text{...(i)}$$



Let E be the potential drop across resistance (R), then

$$E = IR \quad \text{...(ii)}$$

$\because$ Instantaneous emf = Instantaneous value of potential drop

From Eqs. (i) and (ii), we have

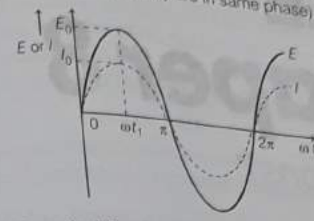
$$IR = E = E_0\sin\omega t$$

$$\Rightarrow \quad I = \frac{E}{R} = \frac{E_0\sin\omega t}{R}$$

$$\Rightarrow \quad I = I_0\sin\omega t \qquad \left[\because I_0 = \frac{E_0}{R}\right] \quad \text{...(iii)}$$

Comparing $I_0 = E_0/R$ with Ohm's law, we find that resistors work equally well for both AC and DC voltages.

From Eqs. (i) and (iii), we get that for resistor there is zero phase difference between instantaneous alternating current and instantaneous alternating emf (i.e. they are in same phase).



(ii) Power is defined as the rate of doing work. [2]

$$P = \frac{dW}{dt} \quad \text{...(i)}$$

Also power is defined as the product of voltage and current.

In AC circuit, both emf and current change continuously with respect to time. So in it we have to calculate average power in complete cycle ($0 \to T$).

Instantaneous power, $P = EI$ ...(ii)

Here, E and I are instantaneous voltage and current, respectively. If the instantaneous power remains constant for a small time dt, then small amount of work done in maintaining the current for a small time dt is

$$\frac{dW}{dt} = EI$$

$$\Rightarrow \quad dW = EI\,dt \quad \text{...(iii)}$$

Integrating Eq. (iii) on both sides, we get

$$\int dW = \int_0^T EI\,dt$$

Total work done in maintaining current in pure R,

$$W = \int_0^T E_0\sin\omega t\, I_0\sin\omega t\,dt$$

$$= E_0I_0\int_0^T \sin^2\omega t\,dt = E_0I_0\int_0^T \frac{(1-\cos 2\omega t)}{2}dt$$

$$= \frac{E_0I_0}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\,dt\right] = \frac{E_0I_0}{2}\{[t]_0^T - 0\}$$

$$\because \int_0^T \cos 2\omega t\,dt = 0$$

$$W = \frac{E_0I_0}{2}T$$

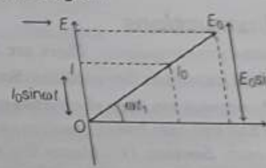
Average power dissipated, $P_{av} = \frac{W}{T}$

$$\Rightarrow \quad P_{av} = \frac{E_0I_0}{2} = E_{rms}\,I_{rms}$$

where, $I_{rms} = \frac{I_0}{\sqrt{2}}$

$$E_{rms} = \frac{E_0}{\sqrt{2}}$$ [2]

(iii) **Phasor Diagram**



Here, peak values E_0 and I_0 are represented by vectors rotating with angular velocity ω with respect to horizontal reference. Their projections on vertical axis give their instantaneous values. [1]