SET-3

Roll No. | | | | | | |

Candidates must write the Set No. on the title page of the answer book.

SAHODAYA PRE BOARD EXAMINATION - 2023-24

- Please check that this question paper contains **07** printed pages.
- Set number given on the right-hand side of the question paper should be written on the title page of the answer book by the candidates.
- Check that this question paper contains **38** questions.
- Write down the Serial Number of the question in the left side of the margin before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed 15 minutes prior to the commencement of the examination. The students will read the question paper only and will not write any answer on the answer script during this period. Students should not write anything in the question paper.

CLASS- XII

SUB : MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks : 70**

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A

All questions are compulsory. In case of internal choices attempt any one.

1. The function $f: N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$, is

(a) Bijective (b) One-one but not Onto

(c) Onto but not one-one (d) Neither one-one nor onto

2. If A = $\begin{bmatrix} \propto & 2 \\ 2 & \propto \end{bmatrix}$ and $|A^3|$= 125 , the value of $\propto$ is

(a) 5 (b) 0 (c) ± 1 (d) ± 3

3. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & -3 \\ 9 & 6 & -2 \end{vmatrix}$ = 0 then the value of x is

(a)3 (b) 5 (c)7 (d) 9

4. The value of $f(0)$ so that $f(x) = \frac{-e^x + 2^x}{x}$ may be continuous at $x = 0$ is

(a) $\log\left(\frac{1}{2}\right)$ (b) 0 (c)4 (d) $-1 + \log 2$

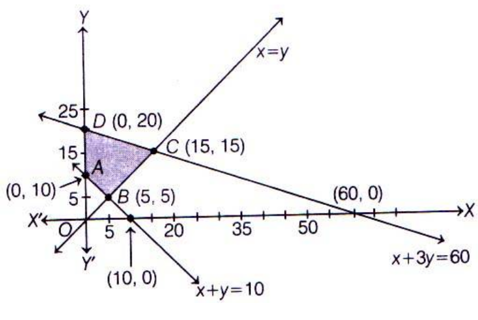
5. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $x = 1, \frac{y+1}{2} = \frac{z-1}{-3}$ are mutually perpendicular then the value of k is

(a) 2 (b) $2/3$ (c) $-2/3$ (d) -2

6. The product of order and degree of the differential equation $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$ is

(a) 3 (b) 4 (c) 2 (d)1

7. Based on the given shaded region as the feasible region in the graph, at which point(s), the objective function Z = 3x+9y is maximum?



(a) Point B (b) Point C (c) Point D (d) Every point on the line segment CD

8. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$,then

(a) $\vec{a} \parallel \vec{b}$ (b) $\vec{a} \perp \vec{b}$ (c) $|\vec{a}| = |\vec{b}|$ (d) None of these

9. If $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, then "$a$" is

(a) 1 (b) 1/2 (c) 3 (d) 2

10. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 + \beta = 0$

(c) $1 - \alpha^2 - \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$

11. Corner points of the feasible region is determined by the system of linear constraints are (0, 3), (1, 1), (3, 0). Let $Z = px + qy$, where $p, q > 0$, condition on p , q so that the minimum of Z occurs at (3, 0) and (1, 1) is

(a) p=2q (b) p=q/2 (c) p=3q (d) p = q

12. If a line makes an angle $\frac{\pi}{3}$ with each X and Y axes then the obtuse angle made with Z axis is

(a) $\frac{3\pi}{2}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{6}$

13. If A is a square matrix of order 3, such that A (adj A) = 10I, then |adj A| is equal to

(a) 1 (b) 10 (c) 100 (d) 101

14.If A and B are two events such that $P(A)$=1/2, $P(B)$=1/3, and P(A/B) =1/4, then $P(A' \cap B')$ is

(a) 1/12 (b) 3/4 (c) 1/4 (d) 3/16

15. If a and b are order and degree of the differential equation

$\left(y\frac{dy}{dx} + x^3\left(\frac{d^2y}{dx^2}\right)\right)^2 + xy = cosx$, then

(a) $a < b$ (b) $a = b$ (c) $a > b$ (d) none of these

16. The direction cosines of vector $\overrightarrow{BA}$, where coordinates of A and B are $(1, 2, -1)$ and $(3, 4, 0)$ respectively, are:

(a) $-2, -2, -1$ (b) $-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ (c) $2, 2, 1$ (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

17. If $y = \sec x^0$, then $\frac{dy}{dx}$ is equal to :

(a) sec x tan x (b) sec x° tan x° (c) $\frac{\pi}{180}$sec x° tan x° (d) None of these

18. The number of lines passing through the origin which make equal angles with the coordinate axes is

(a) 1 (b) 4 (c) 8 (d) 2

ASSERTION-REASON BASED QUESTIONS

In the following two questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** Let $A = \{-1, 1, 2, 3\}$ and $B = \{1, 4, 9\}$, where $f : A \to B$ given by $f(x) = x^2$, then f is a many-one function.

Reason (R) : If $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$, for every $x_1, x_2 \in$ domain, then f is one-one or else many-one.

20. **Assertion (A):** The two positive numbers x and y are such that x+y=35 and $x^2 y^5$ is maximum, then the numbers are 10 and 25.

Reason(R): If f be a function, defined on an interval I and c $\in$ I and also, if f be twice differentiable at c, then x=c is a point of local maximum if $f^{/}(c) = 0$ *and* $f^{//}(c) < o$ and the value of f(c) is local maximum value of f.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Find the domain of $f(x) = \sin^{-1}\sqrt{x-1}$.

OR

Find the value of $\sin(2\tan^{-1}(0.75))$.

22. Find the intervals on which the function f(x) =$2x^3+9x^2+12x+20$ is increasing .

23. Find the maximum value of the function $f(x) = \sin x + \cos x$.

OR

A particle moves along the curve$\frac{x^2}{16} + \frac{y^2}{4} = 1$. Find the co-ordinates of the points, where the rate of change of abscissa is 4 times that of its ordinate. .

24. Evaluate$\int_0^1 x(1-x)^n\, dx$.

25. Divide 64 into two parts such that sum of the cubes of two parts is minimum.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Evaluate:$\int \frac{x^2+x+1}{(x+1)(x^2+1)} dx$.

27. Two defective bulbs are accidentally mixed with 6 good ones. If three bulbs are drawn at random, find the mean of the number of defective bulbs drawn.

28. Evaluate: $\int \{\log(\log x) + \frac{1}{(\log x)^2}\}\, dx$

OR

Evaluate: $\int_0^4 (|x| + |x-2| + |x-4|)\, dx$

29. Solve $\left(1 + e^{x/y}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$

OR

Solve $\frac{dy}{dx} = \sec(x+y)$

30. Solve the following linear programming problem Minimize $Z = 3x + 5y$ subject to constraints.

$x + 3y \geq 3$, $x + y \geq 2$, $x \geq 0, y \geq 0$

OR

Solve the following linear programming problem Maximize $Z = -x + 2y$ subject to constraints.

$-x + 3y \leq 0$, $x + y \leq 6, x - y \leq 2, x \geq 0, y \geq 0$

31. Find the solution of the differential equation: $\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x \ (x \neq 0)$, given that $y = 0$ when $= \pi/2$.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Find the area of the region in the first Quadrant enclosed by X-axis , the line y = x and the curve $x^2 + y^2 = 32$.

33. If A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $6A^{-1} = A^2 + cA + dI$ then find the value of (c, d).

34. Prove that the relation R on the set N×N defined by $(a, b)\ R\ (c, d)$ iff $ad(b + c) = bc(a + d)$ for all $(a, b), (c, d) \epsilon N \times N$ is an equivalence relation.

OR

Let $f: R_+ \rightarrow [-5, \infty)$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that f is bijective.

35. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, write the equation of the line joining the given point and its image and find the length of the segment joining the given point and its image.

OR

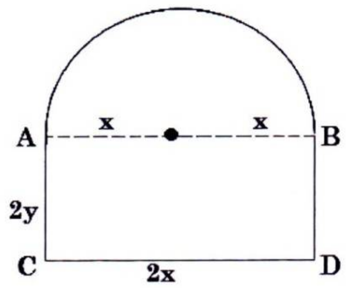
Show that the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect.Also find the equation of a line passing through the point of intersection of the above lines and parallel to z – axis.

SECTION E

(This section comprises of 3 case-study/passage-based questions 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **Case-Study 1**: Read the following passage and answer the questions given below.

Mr Shashi, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different from other floors.The window is in the shape of a rectangle which is surmounted by a semi-circular opening.
This window is having a perimeter of 10 m.



(i) If 2x and 2y represents the length and breadth of the rectangular portion of the windows then, find the value of y, in terms of x.

(ii) Express the combined area (A) of the rectangular region and semi-circular region of the window, as a function of x.

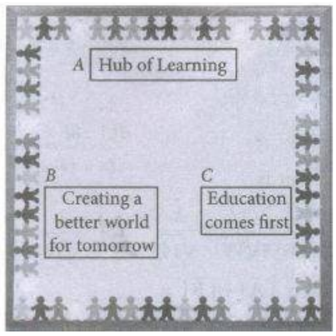
(iii) Find the maximum value of area A of the whole window using derivatives.

OR

(iii) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible. For this to happen, find the length and breadth of rectangular portion of the window. Also find the radius of semi-circular opening of the window. Use derivatives.

37. **Case-Study2:**:

Three slogans on chart papers are to be placed on a school bulletin board at the points A, B and C displaying A (Hub of Learning), B (Creating a better world for tomorrow) and C (Education comes first). The coordinates of these points are (1, 4, 2), (3, -3, -2) and (-2, 2, 6) respectively.



Based on the above situation, answer the following:

(i) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be the position vectors of points A, B and C respectively, then find the value of $\vec{a} + \vec{b} + \vec{c}$.

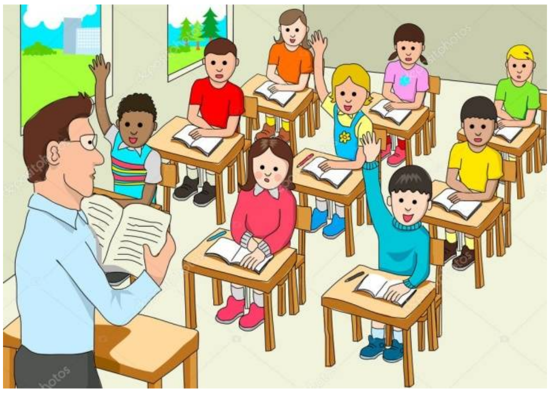
(ii) What will be the area of ΔABC?

(iii) Suppose, if the given slogans are to be placed on a straight line, then find the value of $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$

OR

If $\vec{a} = 2i+3j+6k$ then find the unit vector in the direction of vector $\vec{a}$.

38. **Case-Study 3:** After observing attendance register of Class -XII, Academic committee comes on conclusion that, 30% students have 100% attendance and 70% students are irregular to attend class. When previous year result being observed it was found that 80% of all students who have 100% attendance secured 95% and above in XII Board exam where 10% irregular students have secured 95% and above marks. At the end of the session, one student is chosen at random from the class, then



Basing on the above concept answer the following

(i) Find probability that the selected student has secured 95% and above marks given that the student has 100% attendance. Also Find the probability that selected student has secured 95% and above marks given that the student is irregular.

(i) If by random selection, selected student has secured 95% and above marks, find the probability that the students has100% attendance.

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