SET NO-2

Roll No. | | | | | | | |

Candidates must write the Set No. on the title page of the answer book.

DAV PUBLIC SCHOOLS, ODISHA ZONE
HALF YEARLY EXAMINATION, 2023-24

- Please check that this question paper contains **6** printed pages.
- Set number given on the right hand side of the question paper should be written on the title page of the answer book by the candidate.
- Check that this question paper contains 38 questions.
- Write down the Serial Number of the question in the left side of the margin before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed 15 minutes prior to the commencement of the examination. The students will read the question paper only and will not write any answer on the answer script during this period.

CLASS- XII
SUB: MATHEMATICS

Time: 3 Hours **Maximum Marks:80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section- B carries 10 marks weightage, Section – C carries 18 marks weightage, Section – D carries 20 marks weightage and Section – E carries 3 case-based with total weightage of 12 marks.
   Section – A:
3. It comprises of **20 MCQs of 1 mark** each.
   Section – B:
4. It comprises of **5 VSA type questions of 2 marks** each.
   Section – C:
5. **It comprises of 6 SA type questions of 3 marks** each.
   Section – D:
6. It comprises of **4 LA type questions of 5 marks** each.
   Section – E:
7. It has **3 case studies based questions of 4 marks each with sub parts.**
8. Internal choice is provided in **2 questions in Section – B, questions in Section – C, 2 questions in Section – D.** You have to attempt only one of the alternatives in all such questions.

SECTION – A

(All questions are compulsory. No internal choice is provided in this section)

1. Let A = {1, 2, 3, ……n} and B = {a, b}. Then the number of surjections from A into B is

(a) $^nP_2$ (b) $2^n - 2$ (c) $2^n - 1$ (d) none of these

2. The area of the region bounded by the curve $y = cosx$ between $x = 0$ and $x = \pi$ is

(a) 2 sq units (b) $\frac{3}{2}$ sq units (c) 6 sq units (d) 8 sq units

3. **The area of the region bounded by the $y = x^2$ and the line $y = 16$ is**

(a) $\frac{32}{3}$ sq units b) $\frac{256}{3}$ sq units (c) $\frac{64}{3}$ sq units (d) $\frac{128}{3}$ sq units

4. **The order of differential equation $\frac{d}{dx}(\frac{dy}{dx}) + \sin(\frac{dy}{dx}) = \log x$ is**

(a)1 (b) 2 (c) 3 (d) none of these

5. **The Integrating factor of differential equation $(x – y^3)\, dy + y\, dx = 0$ is**

(a) $\frac{1}{y}$ (b) log y (c) y (d) y^2

6. **The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is**

(a) $e^x + e^{-y} = c$ (b) $e^x + e^y = c$ (c) $e^{-x} + e^y = c$ (d) $e^{-x} + e^{-y} = c$

7. **The number of functions defined from {1, 2, 3} to {a, b, c, d} which are not one-one is**

(a) 24 (b) 64 (c) 81 (d) 40

8. **The value of $sin^{-1}\{\sin\left(-\frac{17\pi}{8}\right)\}$ is**

(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{8}$ (d) $-\frac{\pi}{8}$

9. **The value of $2\tan^{-1}\frac{1}{\sqrt{3}} - \cot^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ is**

(a) $-\frac{5\pi}{12}$ (b) $\frac{\pi}{12}$ (c) $-\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$

10. **If A is a square matrix of order 3 and $|adjA| = 25$ then the value of $3|A|$ is**

(a) 5 (b) ± 15 (c) ± 5 (d) None of these

11. **If the matrix $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew-symmetric, then the value of $x + y$ is**

(a) 3 (b) 0 (c) 2 (d) 4

12. **If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2} =$**

(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3t}{2}$

13. **The interval in which the function f given by $f(x) = x^2e^{-x}$ is strictly increasing is**

(a) $(-\infty, \infty)$ (b) $(-\infty, 0)$ (c) $(2, \infty)$ (d) $(0, 2)$

14. **The sides of an equilateral triangle are changing at the rate of 2cm/sec. The rate at which its area increases, when side is 10cm is**

(a) $10\ cm^2/\sec$ (b) $\sqrt{3}cm^2/\sec$ (c) $10\sqrt{3}cm^2/\sec$ (d) $\frac{10}{3}cm^2/\sec$

15. **If x is real, the minimum value of $x^2 – 8x + 17$ is**

(a) -1 (b) 0 (c) 1 (d) 2

16. $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}\, dx =$

(a) sin x +c (b) cos x + c (c) x + c (d) $x^2 + c$

17. **The value of $\int \frac{dx}{1+e^x}$ is**

(a) $\log_e(1+e^x)+c$ (b) $\log_e(1+e^{2x})+c$

(c) $\log_e(1+e^{-2x})+c$ (d) $-\log_e(1+e^{-x})+c$

18. $\int \cos^{-1}(\sin x)dx$ **is equal to**

a) $\frac{\pi}{2}-x+c$ b) $\frac{\pi x}{2}-\frac{x^2}{2}+c$ (c) $\frac{\pi x}{2}+\frac{x^2}{2}+c$ d) none of these

For Questions 19 and 20, two statements are given – one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv)as given below:

(i) Both A and R are true and R is the correct explanation of the assertion.

(ii) Both A and R are true and R is not the correct explanation of the assertion.

(iii) A is true, but R is false.

(iv) A is false, but R is true

19. Assertion (A): Range of f(x) = $\sin^{-1}x + 2\cos^{-1}x$ is $[0, \pi]$.

Reason (R) : Principal value branch of $\cos^{-1}x$ is $[0, \pi]$.

20. Assertion: The function $f(x)=\frac{x-1}{x-2}$ is discontinuous at x=2.

Reason: Every polynomial function is continuous on domain R.

SECTION –B

(All questions are compulsory. In case of any internal choice, attempt any one question only)

21. Show that $f: R-\left\{-\frac{3}{5}\right\} \to R-\left\{\frac{2}{5}\right\}$ defined by $f(x)=\frac{2x}{5x+3}$ is surjective.

OR

Show that the function f: R → R defined by f(x) = $\frac{x}{1+x^2}$, ∀ x ϵ R is neither one-one nor onto.

22. Find the principal value of $\sin^{-1}\left(\cos\left(\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right)\right)$

23. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2}<x<\frac{\pi}{2}$ in the simplest form.

24. Show that $y=\log(1+x)-\frac{2x}{2+x}, x>-1$, is an increasing function of x throughout its domain.

OR

The total cost $C(x)$ in Rupees, associated with the production of x units of an item is given by $C(x)=0.005x^3-0.02x^2+30x+5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

25. Evaluate $\int \frac{x-3}{(x-1)^3}e^x dx$.

SECTION – C

(All questions are compulsory. In case of any internal choice, attempt any one question only)

26. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{tanx}}$.

27. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right)$ = 3x + 4y, given that y = 0 when x = 0.

OR

Find the general solution of the differential equation $y\,dx - (x + 2y^2)\,dy = 0$.

28. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$

29. If y=$(logx)^x$+$(x)^{cosx}$, then find $\frac{dy}{dx}$

OR

Find the values of a and b such that the function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, if\ x < 4 \\ a + b,\ \ if\ x = 4 \\ \frac{x-4}{|x-4|} + b, if\ x > 4 \end{cases}$$ is a continuous function at x=4.

30. Find the intervals in which the function f(x) = 20 – 9x + 6x^2 – x^3 is strictly increasing or strictly decreasing.

31. Evaluate: $\int \frac{x^2}{(x^2+4).(x^2+9)} dx$.

SECTION – D

(All questions are compulsory. In case of any internal choice, attempt any one question only)

32. Find the values of a and b so that the function $f(x) = \begin{cases} x^2 + 3x + a,\ x \leq 1 \\ bx + 2,\ x > 1 \end{cases}$ is differentiable for all $x \in R$.

33. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, using the result.

Evaluate : $\int_0^{\pi/2} \frac{sin^2x}{sinx+cosx} dx.$

34. Prove that the relation R on the set N×N defined by $(a, b)\ R\ (c, d)$ iff $ad(b + c) = bc(a + d)$ for all $(a, b), (c, d) \epsilon N \times N$ is an equivalence relation.

OR

Consider f: $R^+ \rightarrow [-5, \infty)$ given by f(x) = 9x^2 + 6x – 5. Show that f is one-one and onto.

35. If A = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, Find A^{-1}. Using A^{-1}, solve the system of linear equations x – 2y =10, 2x – y – z = 8, -2y + z = 7.

OR

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

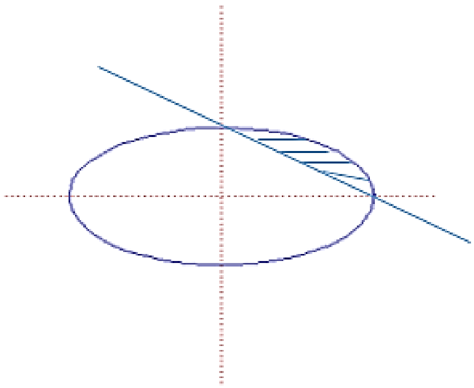
$x - y + 2z = 1, 2y - 3z = 1 \; and \; 3x - 2y + 4z = 2$

SECTION –E

(All questions are compulsory. In case of any internal choice, attempt any one question only)

36. **Case-Study-3**.

A mirror in the shape of an ellipse represented by the



equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$was hanging on the wall. Arun and his sister were playing with a ball inside their house. All of a sudden, the ball hit the mirror and gets a scratch in the shape of a line represented by $\frac{x}{3} + \frac{y}{2} = 1$.

Based on above information, answer the following questions:

(i) Find the point of intersection of the Mirror (Ellipse) and Scratch (Straight Line) and find the value of : $\int_0^3 \left(1 - \frac{x}{3}\right) dx$

(ii) Find the area of the smaller region bounded by the Mirror and Scratch.

37. **Case-Study 1:** Read the text carefully and answer the questions**:** A manufacturer produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below:

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
I	10,000	2,000	18,000
II	6,000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are₹ 2.50, ₹ 1.50 and ₹ 1.00 respectively, Based on the information given above, answer the following questions:

(i) Find the total revenue collected from Market - I .

(ii) Find the total revenue collected from Market - II.

(iii) Find the gross profit from both markets considering the unit costs of the three commodities as ₹ 2.00, ₹ 1.00, and 50 paise respectively.

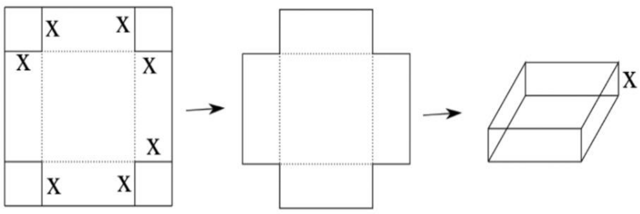
OR

If gross profit from both markets is Rs.1000 , then find |A| +| adj(A)|

if $A(\text{adj}A) = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$

38. **Case-Study-2**. A shopkeeper wants to prepare an open box for keeping the sweets on it. He has a rectangular tin aluminium sheets of size 3 mt. by 8 mt. He remove square pieces of equal size of $x$mt. from each corner to make the required box.

Based on above information, answer the following:



(i) Write the volume function in terms of x.

(ii) Find the length of square piece, when the volume is maximum.

(iii) Find the maximum volume of the box, by using first derivative test.

OR

Find the maximum volume of the box, by using second derivative test.